Disorder effect on the focus image of sonic crystals in air

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(Received 20 February 2006; published 26 May 2006)

When acoustic waves propagate in two-dimensional sonic crystals composed of parallel rigid cylinders in air, anisotropic band gaps, such as a partial gap and deaf band, forbid the waves within certain frequency regions from propagating along certain directions, thus forming a stable imaging focus effect. If the introduced disorder has not destroyed the original anisotropic band gap, this unique effect still exists, although the focused image becomes blurred. Once the sample reaches complete disorder, the anisotropic band gap is destroyed, and this effect also disappears.

DOI: 10.1103/PhysRevE.73.056615 PACS number(s): 43.20.+g, 43.90.+v

I. INTRODUCTION

When an electron, such as a quantum wave, propagates inside a periodic atom array, it is modulated with a periodic structure, thus forming an electronic band structure. Similarly, once a light wave or acoustic wave propagates inside a periodic structure, due to the property of the Bloch wave, a band structure with a band gap also occurs. The corresponding lattice structure is called a photonic crystal [1] or sonic crystal [2–6].

Several years ago, it was predicted that, based on a flat slab photonic crystal, a superlensing phenomenon should be realized within certain frequency regimes [7]. The following year, this focusing image effect was indeed observed across flat photonic crystal slabs. Initially, negative refraction was introduced to explain this superlensing effect, which also yielded a new concept, namely left-handed materials [8]. Meanwhile, because of the similarities between acoustic and light waves, the image superlensing phenomenon also occurs in sonic crystals [9,10]. Here, a new explanation is proposed that, due to the anisotropic band gap of the periodic lattice, a wave is permitted to propagate only in certain directions and frequency regimes, where the so-called wave-guiding or self-collimation effects can be formed.

As mentioned above, an acoustic wave propagating inside a periodic structure produces a band structure, where an anisotropic band gap is a condition for the superlensing effect. Yet when we fabricate a sonic crystal, it is possible to introduce disorder into the periodic structure. Acoustic wave propagation in a random or completely-disordered structure has been independently investigated by Håkansson *et al.* [11] and Hoskinson *et al.* [12]. Recently, Gupta found that the strength of disorders in sonic crystals can affect the robustness of the focusing behavior [13]. Based on these studies, the problems as to what is the effect of the introduced disorder on the superlensing effect and how to discover its underlying mechanism are still open, which is the purpose of the present paper. The resolutions of these problems may pave the way for new applications of sonic crystals, such as ultrasonic imaging and manipulation of acoustic flows.

II. THEORY

Let us assume the formation of a square array of N rigid cylinders with radius $a=1.5$ cm in a sonic crystal, whose lattice constant is $d=11.0$ cm. We then place an acoustic line source transmitting monochromatic waves at a certain spatial point $\vec{r}_s$. As an acoustic wave is submitted by the source, the scattered wave from each cylinder also contributes to the total waves, forming so-called multiply scattering. The final wave reaching a receiver located at $\vec{r}_r$ is the sum of the direct wave from the source and the scattered waves from all the cylinders. Such scattering could be solved exactly, following a method proposed by Twersky [14].

The acoustic intensity field is defined as $|p(\vec{r})|^2$, and the acoustic transmission through the cylinder arrays is normalized as $T=p/p_0$, where $p(\vec{r})$ is the the total wave at any space point and p_0 is the direct incident wave from the source. The total wave incident around the ith scatterer $p_{in}^i(\vec{r})$ is a superposition of the direct contribution from the source $p_0(\vec{r})$ and the scattered waves from all other scatterers:

$$p_{in}^i(\vec{r}) = p_0(\vec{r}) + \sum_{j=1,j\neq i}^{N} p_s(\vec{r},\vec{r}_j). \tag{1}$$

The scattered wave from the jth cylinder can be written as

$$p_s(\vec{r},\vec{r}_j) = \sum_{n=-\infty}^{\infty} i\pi A_n^j H_n^{(1)}(k|\vec{r}-\vec{r}_j|)e^{in\phi_{\vec{r}-\vec{r}_j}}, \tag{2}$$

where A_n^i is the coefficient to be determined, $H_n^{(1)}$ is the nth-order Hankel function of the first kind, and $\phi_{\vec{r}-\vec{r}_j}$ is the azimuthal angle of the vector $\vec{r}-\vec{r}_i$ relative to the positive x axis.

The total incident wave around the ith scatterer, $p_{in}^i(\vec{r})$ from the source, and the scattered waves from all other scat-

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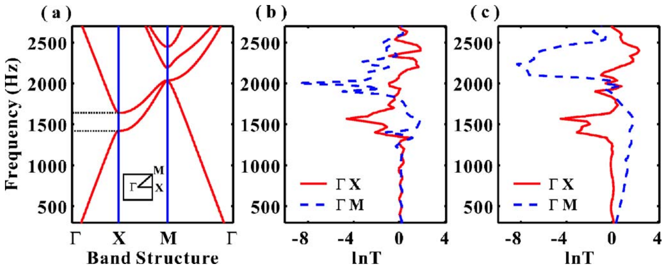

FIG. 1. (Color online) (a) Band structure of a square lattice of rigid cylinders in air, with a partial gap between the two horizontal lines, with the normalized transmission versus frequency for the source inside (b) and outside (c) the sonic crystal.

terers can be expressed in terms of the Bessel function as:

$$p_{in}^{i}(\vec{r}) = \sum_{n=-\infty}^{\infty} B_n^i J_n(k|\vec{r}-\vec{r}_i|)e^{in\phi_{\vec{r}-\vec{r}_i}}. \tag{3}$$

Using the addition theorem for Bessel functions, the scattered waves $p_s(\vec{r},\vec{r}_j)$ for each $j \neq i$ can be expressed in terms of the modes with respect to the ith scatterer, thus giving

$$B_n^i = S_n^i + \sum_{j=1, j\neq i}^{N} C_n^{j,i} \tag{4}$$

with

$$S_l^i = i\pi H_{-l}^{(1)}(k|-il\phi_{\vec{r}_i}|) \tag{5}$$

and

$$C_n^{j,i} = \sum_{l=-\infty}^{\infty} i\pi A_l^j H_{l-n}^{(1)}(k|\vec{r}_i-\vec{r}_j|)\exp[i(l-n)\phi_{\vec{r}_i-\vec{r}_j}]. \tag{6}$$

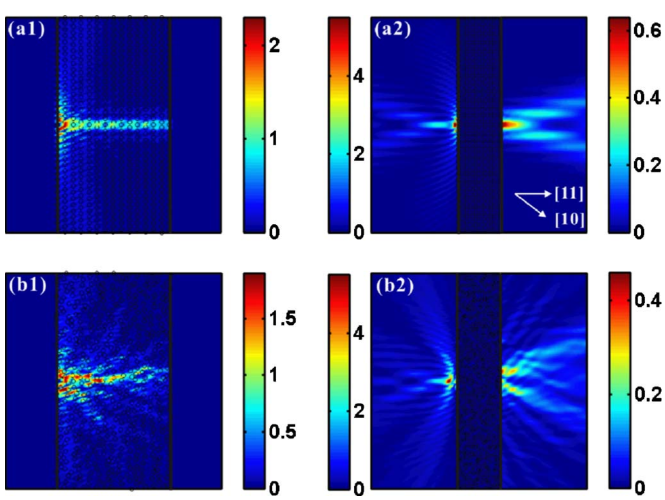

FIG. 2. (Color online) Imaging fields for the frequencies of 1.50 kHz located within the partial gap. (a1) and (a2) show the intensity field inside and outside the sonic crystal, respectively. When a 0.5 disorder degree is introduced into the sonic crystal, (b1) and (b2) plot the intensity fields inside and outside the sample, respectively.

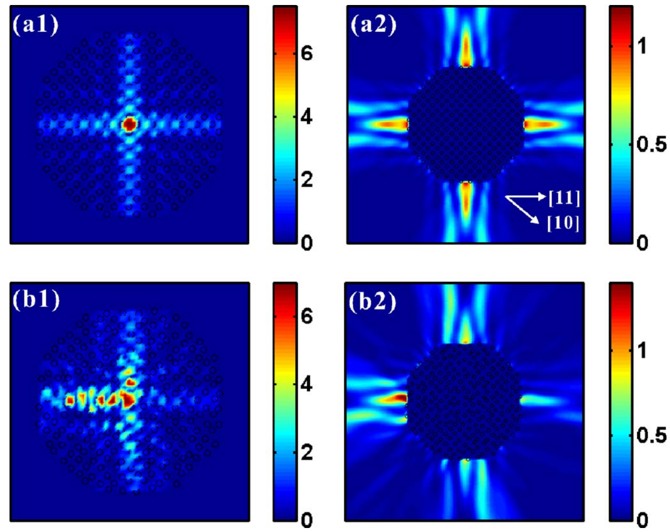

FIG. 3. (Color online) Imaging fields for a transmitting source located inside a round sample consisting of rigid cylinders. (a1) and (a2) show the intensity field inside and outside the sonic crystal, respectively. For a 0.5 disorder degree introduced into the sonic crystal, (b1) and (b2) plot the intensity fields inside and outside the sample, respectively.

Using the usual boundary conditions for each cylinder, a matrix equation can be easily obtained as

$$B_n^i = i\pi\Gamma_n^i A_n^i, \tag{7}$$

where Γ_n^i is a transfer matrix relating the acoustic properties of the scatterers,

$$\Gamma_n^i = \left[\frac{H_n^{(1)}(ka^i)J_n'(ka^i/h^i) - g^i h^i H_n^{(1)\prime}(ka^i)J_n(ka^i/h^i)}{g^i h^i J_n'(ka^i)J_n(ka^i/h^i) - J_n(ka^i)J_n'(ka^i/h^i)}\right]. \tag{8}$$

Thus, the total wave at any desired point outside the cylinders can be obtained as

$$p(\vec{r}) = p_0(\vec{r}) + \sum_{i=1}^{N}\sum_{n=-\infty}^{\infty} i\pi A_n^i H_n^{(1)}(k|\vec{r}-\vec{r}_i|)e^{in\phi_{\vec{r}-\vec{r}_i}}. \tag{9}$$

The acoustic intensity is the square module of the transmitted wave where the normalized transmission is given as $T \equiv p/p_0$.

In order to describe the disorder effect on the scattering process, we introduce two new concepts: One is the filling factor $f_s = \pi(a/d)^2 = 0.05842$, which is defined as the area occupation of the cylinders (per unit area), and the other is the disorder factor x denoting the disorder degree, where x ranges from 0 to 1, with $x=0$ and $x=1$ corresponding to the periodic case and complete randomness, respectively. We define the maximal disorder amplitude as $L=(d-2.0\times a)/2.0$. The disorder factor $\Delta = x \times L$ then denotes the random amplitude, where x indicates the disorder degree ranging from 0 to 1, with $x=0$ and $x=1$ corresponding to the periodic case and completely randomness, respectively.

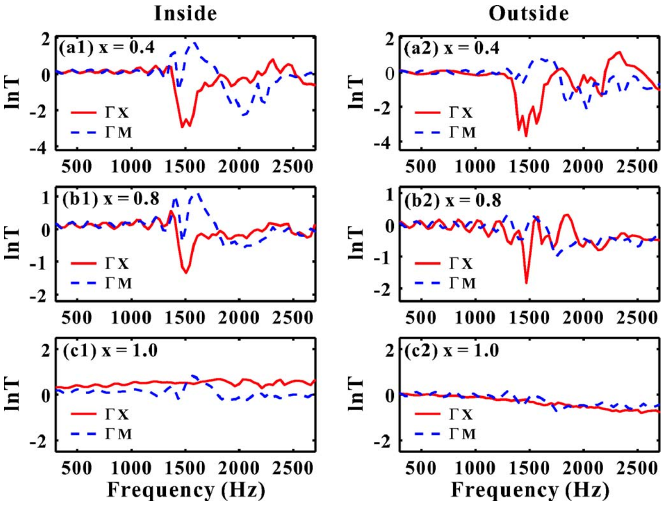

FIG. 4. (Color online) Transmission along the ΓX and ΓM directions when disorder is introduced, corresponding to the transmissions when the source is inside and outside the sample, respectively. (a1) and (a2) show the transmissions of the sample with a 0.5 disorder degree. (b1) and (b2) show the transmissions of the sample with a 0.8 disorder degree. (c1) and (c2) show the transmissions of the completely-disordered sample.

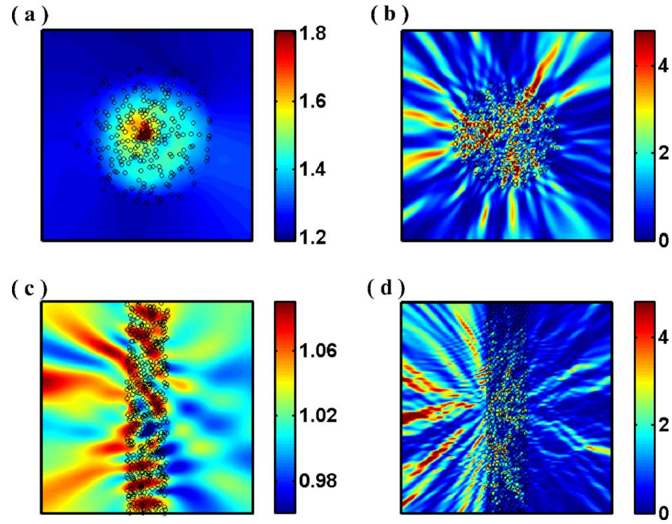

FIG. 5. (Color online) Imaging fields for a completely-disordered sample. (a) and (c) correspond to the imaging fields of a 1.50 kHz acoustic wave for a transmitting source inside and outside a completely disordered sample, respectively. (b) and (d) correspond to the imaging fields of a 2.11 kHz acoustic wave for a transmitting source located inside and outside the completely-disordered sample, respectively.

III. RESULTS AND DISCUSSION

When an acoustic wave propagates inside a lattice of square rigid cylinders, due to its Bloch wave property, the band structure in the first Brillouin zone is well formed, as shown in Fig. 1(a). In the first Brillouin zone, it is apparent that the various dispersion bands lead to anisotropic band gaps along the ΓX and ΓM directions. We designate the frequency range from 1.42 to 1.64 kHz as the partial band gap and the region from 2.08 to 2.72 kHz as the deaf band. Within the partial gap, waves are prohibited from propagating along the ΓX direction, i.e., the [1,0] direction. Similarly, for frequencies inside the deaf band, propagation is inhibited along the [1,1] direction. Transmissions for the source inside and outside the sonic crystal are shown in Figs. 1(b) and 1(c), respectively. We see a small shift of the forbidden propagation regimes caused by the boundary effect, where the transmissions just reflect the anisotropic band gaps. For the acoustic waves in just certain directions, the superlensing effect can be caused by either the partial band gaps or the deaf bands, which can self-guide the wave propagation into passing band directions [9]. Since the focused imaging phenomena are qualitatively similar for both the partial gap and the deaf band, we select one case—the 1.50 kHz acoustic wave within the partial band gap—to explore the disorder effect on the superlensing phenomenon.

For acoustic wave transmission inside a sonic crystal, we construct a slab of size $9d \times 49d$ and place the acoustic source at one lattice constant away from the left side of the slabs. By virtue of the partial gap region along ΓX shown in Fig. 1(a), acoustic waves are prohibited from propagation along the [1,0] direction inside the sonic crystal. Thus, once the source provides a 1.50 kHz acoustic wave, the wave is transmitted along [1,1], as shown in Fig. 2(a1). A well-focused image point, shown in Fig. 2(a2), is formed at the other side of the slab. When 0.5 disorder degree is introduced into the slab, the perfect periodic structure is partly destroyed. More importantly, the submitted acoustic wave still can be focused at the other side of the slab [Fig. 2(b2)], although strictly speaking, the waves cannot propagate along the [1,1] direction [Fig. 2(b1)].

To more clearly exhibit these observations, we place a transmitting source inside the sonic crystal, whose shape is round with a $9d$ radius. Once the source is ignited, the acoustic wave propagates in the [1,1], [−1,1], [−1,−1], and [1,−1] directions, resulting in four images outside the crystal along these four directions, as depicted in Figs. 3(a1) and 3(a2). For the case where the source is inside sample, the introduced 0.5 disorder degree does not destroy the focused images in the four directions [see Fig. 3(b2)], yet the transmission channel is not along a straight line in the original four directions.

When a disorder degree is introduced into the sonic crystal, we choose two cases—the source inside and outside the sample—to show the wave transmission properties. In these two cases, comparing Figs. 1(b) and 1(c), while the introduced disorder degree of the sample has reached 0.4, two forbidden propagation regimes corresponding to the original partial band gap and deaf band still exist, as shown in Figs. 4(a1) and 4(a2). As the disorder degree increases and approaches 0.8, the forbidden propagation of the original partial gap becomes weaker and slowly disappears, as shown in Figs. 4(b1) and 4(b2). As mentioned earlier, since anisotropic band gaps are the conditions for the superlensing effect, it becomes clear that when the disorder degree does not completely destroy the original anisotropic gaps, such as the partial gap and deaf band, the superlensing phenomenon still occurs. Naturally, once the system reaches a completely-disordered state, the two forbidden propagation regimes are destroyed and vanish, as shown in Figs. 4(c1) and 4(c2).

Thus, when the source is fixed at $1d$ away from the left side of a completely-disordered sample, the 1.50 kHz and 2.11 kHz acoustic waves within the original deaf band cannot form a focused image due to the lack of anisotropic gaps, as shown in Figs. 5(a) and 5(b), respectively. The same as above, this specific effect also cannot occur, even though the 1.50 kHz and 2.11 kHz line acoustic sources are placed inside the completely-disordered sample, as shown in Figs. 5(c) and 5(d).

IV. SUMMARY

In summary, due to the anisotropic gap structure, an acoustic wave is forbidden to propagate in certain directions and frequency regions, leading to a stable imaging focus effect. If an introduced disorder does not destroy the forbidden propagation regimes caused by the original anisotropic gaps, such as the partial gap and deaf band, the focusing effect still exists, while the focused image becomes blurred. Once a sample reaches compete disorder, the forbidden propagation regimes are destroyed, and this effect also disappears.

ACKNOWLEDGMENTS

We greatly acknowledge Professor Zhen Ye and Professor Peter Handel for valuable discussions. This work was supported by the U. S. Army Research Office under Contract No. W911NF-04-1-038 and the National Science Foundation of China under Grant No. 90403110.

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